**Section 4.2—Logarithmic Functions**

No horizontal line can be drawn that intersects the graph of an exponential function at more than one point. This means that the exponential function is one-to-one and has an inverse.

The inverse function of the exponential function with base b is called the logarithmic function with base b.

**Logarithmic Function**

For x > 0 & b > 0, b ≠ 1

is equivalent to .

The function is the logarithmic function with base b.

and are two was of expressing the same thing.

* is the **logarithmic form**
* is the **exponential form**

**Example**—Write each equation in its equivalent exponential form.



**Example**—Write each equation in its equivalent logarithmic form.



Remember logarithms another way to write exponents. So if we want to evaluate we have to ask “what exponent must we raise b to in order to get x.”

**Example**—Evaluate.

**Basic Logarithmic Properties Involving One**

* ; 1 is the exponent to which b must be raised to obtain b
* ; 0 is the exponent to which be must be raised to obtain 1

**Example**—Evaluate.

**Inverse Properties of Logarithms**

For b > 0 & b ≠ 1

The logarithm with base b of b raised to a power that

equals that power.

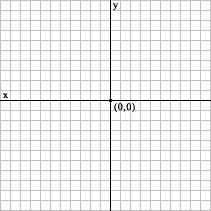
b raised to the logarithm with base b of a number equals

that number.

**Example**—Evaluate



**Graphing a Logarithmic Function**—Remember that log functions are inverses of exponential functions; meaning that the log function reverses the coordinates of the exponential function. It also means the graph of the log is a reflection of the graph of the exponential function about the line y = x.



**Example**—Graph on the same rectangular coordinate system.

The domain of a logarithmic function in the form of is the set of all positive real numbers.

The domain of consists of all x for which g(x) > 0.

**Example**—Find the domain of .

**Common Logarithmic Function**—logarithmic function with base 10

The function is usually expressed as

Many real-life phenomena start with rapid growth and then the growth begins to level off.

**Example**—The percentage of adult height attained by a boy who is x years old can be modeled by where x represents the boys’ age and represents the percentage of his adult height. Approximately what percentage of his adult height has a boy attained at age 10.

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| **Properties of Common Logarithms** | |
| **General Properties** | **Common Log Properties** |
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The property can be used to evaluate common logarithms involving powers of 10. For example:

**Natural Logarithmic Function**—the logarithmic function with base e. The function is usually expressed as .

Like the domain of all logarithmic functions, the domain of the natural logarithmic function is the set of all positive real numbers. Thus, the domain consists of all x for which

**Example**—Find the domain of each function.

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| **Properties of Common Logarithms** | |
| **General Properties** | **Natural Log Properties** |
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**Example**—When the outside air temperature is anywhere from 72° to 96° Fahrenheit, the temperature in an enclosed vehicle climbs by 43° in the first hour. The function models the temperature increase, , in degrees Fahrenheit, after x minutes. Use the function to find the temperature increase, to the nearest degree, after 30 minutes.